ficient to take $n = 1000$; we see that a much smaller number, 500, is also sufficient (and necessary). Part (*b*) of the problem was originally suggested to the proposer by the fact that when the values of $S_1, S_2, S_3, \cdots$ are plotted on a line, two successive points lie on opposite sides of S and about equally distant from S. The midpoint between S_n and S_{n+1} is S_n'. The value S_n' as an approximation to S may also be obtained as follows: We may write

$$
\begin{aligned}
S &= S_n + (-1)^n\left[\frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+3} - \cdots\right] \\
&= S_n + \frac{(-1)^n}{n+1}\left[\frac{1}{(n+1)(n+2)} + \frac{1}{(n+3)(n+4)} + \cdots\right] \\
&= S_n + \frac{(-1)^n}{n+1} - (-1)^n\left[\frac{1}{(n+2)(n+3)} + \frac{1}{(n+4)(n+5)} + \cdots\right].
\end{aligned}
$$

Adding the last two equations and dividing by 2, we have

$$S = S_n' + (-1)^n\left[\frac{1}{(n+1)(n+2)(n+3)} + \frac{1}{(n+3)(n+4)(n+5)} + \cdots\right].$$

If we compare this with the second equation we find

$$|S - S_n'| < \frac{1}{n+3}\cdot|S - S_n| \quad \text{or} \quad |S - S_n'| < \frac{1}{2n(n+3)},$$

which is a slightly better test than that given in the problem. In particular it follows that $|S - S_{21}'| < .001$.

407. Proposed by E. B. ESCOTT, University of Michigan.

In computing the values of the natural logarithms of 2, 3, and 5 by the following formulas:

$$
\begin{aligned}
\log 2 &= 2(7P + 5Q + 3R), \\
\log 3 &= 2(11P + 8Q + 5R), \\
\log 5 &= 2(16P + 12Q + 7R),
\end{aligned}
$$

where P, Q, and R are numbers which were computed by infinite series (G. Chrystal, Algebra, Part II, chapt. 28), it is found, on comparing the results with the known values of these logarithms to 15 decimals, that there are the following errors: -2533, -4052, and 6080, respectively. Find the errors in P, Q and R.

Solution by S. A. Joffe, New York City.

Denoting the computed values of P, Q and R by the same capital letters with primes, and the errors of computation by the corresponding small letters, we will have: $P' = P + p$; $Q' = Q + q$; $R' = R + r$.

Since $\log 2 = 2(7P + 5Q + 3R) = 2[7(P' - p) + 5(Q' - q) + 3(R' - r)]$, and the computed value $\log' 2 = 2(7P' + 5Q' + 3R')$, we see that the error in $\log' 2$, when compared with the known value of $\log 2$, is $\log' 2 - \log 2$, or

$$2(7p + 5q + 3r) = -2533; \tag{1}$$

similarly:

$$2(11p + 8q + 5r) = -4052, \tag{2}$$

$$2(16p + 12q + 7r) = 6080. \tag{3}$$

This system of simultaneous equations may be solved as follows:

Multiplying (1), (2), and (3) by 4, -1, and -1 respectively, we eliminate q and r, and obtain $2p = -4 \times 2533 + 4052 - 6080$, or $p = -6080$.

Multiplying (1), (2), and (3) by -3, -1, and 2 respectively, we eliminate and p, and obtain $2q = 3 \times 2533 + 4052 + 2 \times 6080$, or $q = 11{,}905\frac{1}{2}$.

Multiplying (1), (2), and (3) by -4, 4, and -1, respectively, we eliminate p and q, and obtain $2r = 4 \times 2533 - 4 \times 4052 - 6080$, or $r = -6078$.

Hence the errors in computing P, Q, and R are respectively -6080; 11905.5 and -6078.

Excellent solutions were also received from WALTER C. EELLS, J. W. CLAWSON, and the PROPOSER.

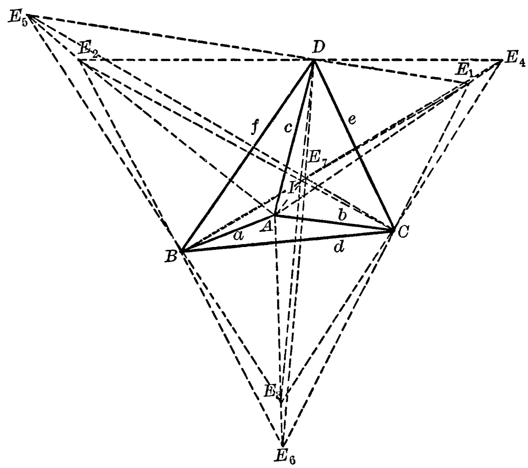
GEOMETRY.

417. Proposed by R. P. BAKER, University of Iowa.

Enumerate the points in which the twelve dihedral bisector planes of a tetrahedron meet, find their multiplicity and account for the 220 points which 12 planes in general determine.

SOLUTION BY J. W. CLAWSON, Collegeville, Pa.

Call the planes bisecting the dihedral angles internally by the letters a, b, c, d, e, f; the planes perpendicular to these which bisect the dihedral angles externally a', b', c', d', e', f'.



Six of these planes pass through the vertex A. Six planes in general determine 20 points. Now a, b, c intersect in the straight line joining A to the center of the inscribed sphere and the opposite ex-center; a', b, c' intersect in a line through A and two of the centers of escribed spheres; a, b', c' in a line joining A to another pair of ex-centers; a', b', c in another such line. These 20 points then reduce to a vertex, taken 16 times, and 4 straight lines passing through that vertex. Since there are four vertices, we can thus account for 80 of the 220 points.

Six of the 12 planes pass through the center of the inscribed sphere. Six planes in general determine 20 points. However a, b, c intersect in the straight line AI; and similarly the three planes passing through each of the other three vertices determine 3 lines. This eliminates 4 of the 20 points from consideration,